less than n elements. Let $A' = a_1, a'_1, \ldots, a'_p, a_2, A_1$ and $A'' = a_2, a''_1, \ldots, a''_q, a_1, A_2$ be two arbitrary ordered sequences of the same n elements, where we have chosen a notation to indicate where the first elements in the sequences a_1 and a_2 respectively occurs in the other sequence. (A_1 and A_2 denote sequences which need not be specified further.) By the ordering assumption a_1Ra_2, a_2Ra_1, $a_2Ra'_i$, a'_iRa_2, $i = 1,\ldots,p$ and $a_1Ra''_j$, a''_jRa_1, $j = 1,\ldots,q$. By successively interchanging a_2 with $a'_p,\ldots,a'_1,a_1$ we obtain the (ordered) sequence $a_2, a_1, a'_1, \ldots, a'_p, A_1$. The two sequences $a'_1,\ldots,a'_p,A_1$ and $a''_1,\ldots,a''_q,A_2$ are two ordered sequences of the same $n-2$ elements. Thus, by the induction assumption, the latter can be obtained from the former by successively permuting consecutive elements. This is then true also for the two sequences $a_2,a_1,a'_1,\ldots,a'_p,A_1$ and $a_2,a_1,a''_1,\ldots,a''_q,A_2$. Finally, by interchanging a_1 successively with $a''_1,\ldots,a''_q$ we obtain $a_2,a''_1,\ldots,a''_q,a_1,A_2$. Thus, the statement is true also for sets of n elements. □

Theorem II.3:28. Suppose that $(E_0, -, T, p)$ is a stochastic event structure and that $|$ is a relation between elements in E_0 satisfying 1) and 2) of definition II.3:24 and such that

1) If $(e_1,\ldots,e_m|e'_1,\ldots,e'_k,e'_{k+1},\ldots,e'_n)$ is in T and $e'_{k+1}|e'_k$, then $(e_1,\ldots,e_m|e'_1,\ldots,e'_{k+1},e'_k,\ldots,e'_n)$ is in T and

$$p(e_1,\ldots,e_m|e'_1,\ldots,e'_k,e'_{k+1},\ldots,e'_n) = p(e_1,\ldots,e_m|e'_1,\ldots,e'_{k+1},e'_k,\ldots,e'_n)$$

2) If $(e_1,\ldots,e_k,e_{k+1},\ldots,e_m|e'_1,\ldots,e'_n)$ is in T and $e_{k+1}|e_k$, then $(e_1,\ldots,e_{k+1},e_k,\ldots,e_m|e'_1,\ldots,e'_n)$ is in T and

$$p(e_1,\ldots,e_k,e_{k+1},\ldots,e_m|e'_1,\ldots,e'_n) = p(e_1,\ldots,e_{k+1},e_k,\ldots,e_m|e'_1,\ldots,e'_n).$$

Then $(E_0, -, T, p, |)$ is a linearly time-ordered stochastic event structure.

Proof. Conditions 1) and 2) of definition II.3:24 for $(E_0, -, T, p, |)$ is satisfied by assumption and condition 3) follows from assumptions 1) and 2) by applying lemma II.3:27. □

Theorem II.3:29. Let $(E_0, -, T, p, t)$ be a stochastic event structure with time and $e_1|e_2$ be the relation $t(e_1) \leq t(e_2)$. Then $(E_0, -, T, p, |)$ is a linearly time-ordered stochastic event structure.

Proof. Conditions 1) and 2) of definition II.3:24 follow from conditions 1) and 2) of definition II.3:20. Condition 3) of definition II.3:24 follows from conditions 3) and 4) of definition II.3:20 by applying theorem II.3:28 (or from the discussion following definition II.3:20). □

II.4 Space and Spacetime Localization of Events

Definition II.4:30. By a stochastic event structure with time and instantaneous space localization we shall mean a structure $(E_0, -, T, p, t, \subset)$ such that

1) $(E_0, -, T, p, t)$ is a stochastic event structure with time.
2) $\subset$ is a relation $e \subset R$ between elements e in E_0 and subsets R of $\mathbb{R}^3$ such that
   a) $e \subset R$ implies $-e \subset R$
   b) $e \subset R \subset R'$ implies $e \subset R'$
   c) $e \subset R$ and $e \subset R'$ implies $e \subset R \cap R'$.

An event e with $e \subset R \subset \mathbb{R}^3$ and $t(e) = t$ can be considered as localized to the "space-time"-region $R' = R \times \{t\} \subset \mathbb{R}^3 \times \mathbb{R}^1 = \mathbb{R}^4$ and we shall write $e \subset R'$

Definition II.4:31. By space-time we shall mean $\mathbb{R}^4$ considered as $\mathbb{R}^3 \times \mathbb{R}^1$. For a point $X = (x,t) = \big((x_1,x_2,x_3),t\big) = (x_1,x_2,x_3,t)$ in space-time, $x = (x_1,x_2,x_3)$ will be called the space coordinates or components and t will be called the time component or simply the time. We define a time-ordering relation $|$ on $\mathbb{R}^4$ in two different ways:

a) "pure time-ordering": for $X' = (x',t')$ and $X'' = (x'',t'')$ in $\mathbb{R}^4$ we define $X'|X''$ to mean $t' \leq t''$
b) "relativistic time-ordering": for $X' = (x'_1,x'_2,x'_3,t')$ and $X'' = (x''_1,x''_2,x''_3,t'')$ in $\mathbb{R}^4$ we define $X'|X''$ to mean that not both

$$(x'_1 - x''_1)^2 + (x'_2 - x''_2)^2 + (x'_3 - x''_3)^2 - (t' - t'')^2 \leq 0$$

and

$$t' > t''$$

In both cases a) and b) we define $R'|R''$ where R' and R'' are subsets of $\mathbb{R}^4$, to mean that $X'|X''$ for any points X' in R' and X'' in R''.

Definition II.4:32. By a space-time localized stochastic event structure we shall mean a structure $(E_0, -, T, p, |, \subset)$ such that

1) $(E_0, -, T, p, |)$ is a general or linearly time-ordered stochastic event structure.
2) $\subset$ is a relation between elements e in E_0 and subsets R of $\mathbb{R}^4 = \mathbb{R}^3 \times \mathbb{R}^1$ such that
   a) $e \subset R$ implies $-e \subset R$
   b) $e \subset R \subset R'$ implies $e \subset R'$
   c) $e \subset R$ and $e \subset R'$ implies $e \subset R \cap R'$
3) $e_1|e_2$ iff $e_1 \subset R_1$ and $e_2 \subset R_2$ for some subsets R_1 and R_2 of $\mathbb{R}^4$ with $R_1|R_2$ where the latter $|$-relation is given by definition II.4:31 a) or b).

There should be no risk of confusion in using the same symbol $|$ for the relation $e_1|e_2$ and $R_1|R_2$.

By the argument following definition II.4:30 a stochastic event structure with time and instantaneous space localization can in a natural way be considered as a space-time localized stochastic event structure with pure time-ordering. On the