less than \( n \) elements. Let \( A' = a_1, a'_1, \ldots, a'_p, A_1 \) and \( A'' = a_2, a''_1, \ldots, a''_q, A_2 \) be two arbitrary ordered sequences of the same \( n \) elements, where we have chosen a notation to indicate where the first elements in the sequences \( a_1 \) and \( a_2 \) respectively occur in the other sequence. \((A_1 \text{ and } A_2 \text{ denote sequences which need not be specified further.})\) By the ordering assumption \( a_1Ra_2, a_2Ra_1, a'_1Ra_2, a'_2Ra_2, i = 1, \ldots, p \) and \( a_1Ra''_j, a''_jRa_1, j = 1, \ldots, q \). By successively interchanging \( a_2 \) with \( a'_2, \ldots, a'_1, a_1 \) we obtain the (ordered) sequence \( a_2, a_1, a'_1, \ldots, a'_p, A_1 \). The two sequences \( a'_1, \ldots, a'_p, A_1 \) and \( a''_1, \ldots, a''_q, A_2 \) are two ordered sequences of the same \( n-2 \) elements. Thus, by the induction assumption, the latter can be obtained from the former by successively permuting consecutive elements. This is then true also for the two sequences \( a_2, a_1, a'_1, \ldots, a'_p, A_1 \) and \( a_2, a_1, a''_1, \ldots, a''_q, A_2 \). Finally, by interchanging \( a_1 \) successively with \( a''_1, \ldots, a''_q \) we obtain \( a_2, a''_1, \ldots, a''_q, a''_1, A_2 \). Thus, the statement is true also for sets of \( n \) elements.

\textbf{Theorem II.3:28.} Suppose that \( (E_0, -, T, p) \) is a stochastic event structure and that \( | \) is a relation between elements in \( E_0 \) satisfying 1) and 2) of definition II.3:24 and such that

1) If \( (e_1, \ldots, e_m | e'_1, \ldots, e'_k, e'_{k+1}, \ldots, e'_n) \) is in \( T \) and \( e'_k | e'_k \)

and \( (e_1, \ldots, e_m | e'_1, \ldots, e'_{k+1}, e'_k, \ldots, e'_n) \) is in \( T \) and

\[ p(e_1, \ldots, e_m | e'_1, \ldots, e'_k, e'_{k+1}, \ldots, e'_n) = p(e_1, \ldots, e_m | e'_1, \ldots, e'_k, \ldots, e'_n) \]

2) If \( (e_1, \ldots, e_k, e_{k+1}, \ldots, e_m | e'_1, \ldots, e'_n) \) is in \( T \) and \( e_{k+1} | e_k \), then

and \( (e_1, \ldots, e_k, e_{k+1}, \ldots, e_m | e'_1, \ldots, e'_n) \) is in \( T \) and

\[ p(e_1, \ldots, e_k, e_{k+1}, \ldots, e_m | e'_1, \ldots, e'_n) = p(e_1, \ldots, e_k, e_{k+1}, \ldots, e_m | e'_1, \ldots, e'_n) \].

Then \( (E_0, -, T, p, |) \) is a linearly time-ordered stochastic event structure.

\textit{Proof.} Conditions 1) and 2) of definition II.3:24 for \( (E_0, -, T, p, |) \) is satisfied by assumption and condition 3) follows from assumptions 1) and 2) by applying lemma II.3:27.

\textbf{Theorem II.3:29.} Let \( (E_0, -, T, p, t) \) be a stochastic event structure with time and \( e_1 | e_2 \) be the relation \( t(e_1) \leq t(e_2) \). Then \( (E_0, -, T, p, |) \) is a linearly time-ordered stochastic event structure.

\textit{Proof.} Conditions 1) and 2) of definition II.3:24 follow from conditions 1) and 2) of definition II.3:20. Condition 3) of definition II.3:24 follows from conditions 3) and 4) of definition II.3:20 by applying theorem II.3:28 (or from the discussion following definition II.3:20).

\section*{II.4 Space and Spacetime Localization of Events}

\textbf{Definition II.4:30.} By a stochastic event structure with time and instantaneous space localization we shall mean a structure \( (E_0, -, T, p, t, \subset) \) such that
1) \((E_0, -, T, p, t)\) is a stochastic event structure with time.

2) \(\subset\) is a relation \(e \subset R\) between elements \(e\) in \(E_0\) and subsets \(R\) of \(\mathbb{R}^3\) such that
   a) \(e \subset R\) implies \(-e \subset R\)
   b) \(e \subset R \subset R'\) implies \(e \subset R'\)
   c) \(e \subset R\) and \(e \subset R'\) implies \(e \subset R \cap R'\).

An event \(e\) with \(e \subset R \subset \mathbb{R}^3\) and \(t(e) = t\) can be considered as localized to the “space-time”-region \(R' = R \times \{t\} \subset \mathbb{R}^3 \times \mathbb{R}^1 = \mathbb{R}^4\) and we shall write \(e \subset R'\)

**Definition II.4:31.** By space-time we shall mean \(\mathbb{R}^4\) considered as \(\mathbb{R}^3 \times \mathbb{R}^1\). For a point \(X = (x, t) = ((x_1, x_2, x_3), t) = (x_1, x_2, x_3, t)\) in space-time, \(x = (x_1, x_2, x_3)\) will be called the space coordinates or components and \(t\) will be called the time component or simply the time. We define a time-ordering relation \(|\) on \(\mathbb{R}^4\) in two different ways:
   a) “pure time-ordering”: for \(X' = (x', t')\) and \(X'' = (x'', t'')\) in \(\mathbb{R}^4\) we define \(X' | X''\) to mean \(t' \leq t''\)
   b) ”relativistic time-ordering”: for \(X' = (x'_1, x'_2, x'_3, t')\) and \(X'' = (x''_1, x''_2, x''_3, t'')\) in \(\mathbb{R}^4\) we define \(X' | X''\) to mean that not both
      \[(x'_1 - x''_1)^2 + (x'_2 - x''_2)^2 + (x'_3 - x''_3)^2 - (t' - t'')^2 \leq 0\]
      and
      \[t' > t''\]

In both cases a) and b) we define \(R' | R''\) where \(R'\) and \(R''\) are subsets of \(\mathbb{R}^4\), to mean that \(X' | X''\) for any points \(X'\) in \(R'\) and \(X''\) in \(R''\).

**Definition II.4:32.** By a space-time localized stochastic event structure we shall mean a structure \((E_0, -, T, p, |, \subset)\) such that

1) \((E_0, -, T, p, |)\) is a general or linearly time-ordered stochastic event structure.

2) \(\subset\) is a relation between elements \(e\) in \(E_0\) and subsets \(R\) of \(\mathbb{R}^4 = \mathbb{R}^3 \times \mathbb{R}^1\) such that
   a) \(e \subset R\) implies \(-e \subset R\)
   b) \(e \subset R \subset R'\) implies \(e \subset R'\)
   c) \(e \subset R\) and \(e \subset R'\) implies \(e \subset R \cap R'\)

3) \(e_1 | e_2\) iff \(e_1 \subset R_1\) and \(e_2 \subset R_2\) for some subsets \(R_1\) and \(R_2\) of \(\mathbb{R}^4\) with \(R_1 | R_2\) where the latter \(|\) relation is given by definition II.4:31 a) or b).

There should be no risk of confusion in using the same symbol \(|\) for the relation \(e_1 | e_2\) and \(R_1 | R_2\).

By the argument following definition II.4:30 a stochastic event structure with time and instantaneous space localization can in a natural way be considered as a space-time localized stochastic event structure with pure time-ordering.