Secure a zone with robots

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Problem. We consider $n$ underwater robots $\mathcal{R}_1, \ldots, \mathcal{R}_n$ at positions $a_1, \ldots, a_n$ and moving in a 2D world [1]. Each robot has a visibility zone. If an intruder is inside the visibility zone of one robot, it is detected. The robots have to collaborate to guarantee that there is no moving intruder inside a subzone of a compact subset $O$ of $\mathbb{R}^2$, representing the 2D ocean.

Complementary approach. We assume that there exists a virtual intruder moving inside $O$ satisfying the differential inclusion

$$\dot{x}(t) \in F(x(t)),$$

where $x(t)$ is the state vector. Moreover, we assume that each robot $\mathcal{R}_i$ has a visibility zone of the form $g_{a_i}^{-1}([0, d])$ where $d$ is the scope. Our contribution is to show that characterizing the secure zone translates into a set-membership set estimation problem [2] where $x(t)$ is shown to be inside the set $X(t)$ returned by our set-membership observer. Then we conclude that $x(t)$ cannot be inside the complementary of $X(t)$. This result can be formalized by the following theorem.

Theorem. The virtual intruder has a state vector $x(t)$ inside the set

$$X(t) = O \cap dt \cdot F(X(t) - dt) \cap \bigcap_i g_{a_i(t)}^{-1}([d(t), \infty]),$$

where $X(0) = O$. As a consequence, the secure zone is

$$S(t) = \text{proj}_{\text{world}}(X(t)).$$
Proof (sketch). Two cases should be considered. If no actual intruder exists then $S(t)$ cannot is secured. If the virtual intruder is a real one, its state $x(t)$ is inside $X(t)$ and its position (which is a part of the state) is inside $\text{proj}_{\text{world}}(X(t))$. In both situations, the intruder can not be inside $S(t)$.

Method. Each robot follows a reference point. All reference points form a flat ellipsoid which plays the role of a barrier. The strategy is illustrated by Figure 1 for 10 robots. The set $\emptyset$ corresponds to the blue area (left). On the right, $S(t)$ is painted green. The observer has been implemented using interval analysis.

References:
