Secure a zone with robots

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Problem. We consider n underwater robots $\mathcal{R}_1, \ldots, \mathcal{R}_n$ at positions $\mathbf{a}_1, \ldots, \mathbf{a}_n$ and moving in a 2D world [**1**]. Each robot has a visibility zone. If an intruder is inside the visibility zone of one robot, it is detected. The robots have to collaborate to guarantee that they is no moving intruder inside a subzone of a compact subset $\mathbb{O}$ of $\mathbb{R}^2$, representing the 2D ocean.

Complementary approach. We assume that there exists a virtual intruder moving inside $\mathbb{O}$ satisfying the differential inclusion

$$\dot{\mathbf{x}}(t) \in \mathbb{F}(\mathbf{x}(t)),$$

where $\mathbf{x}(t)$ is the state vector. Moreover, we assume that each robot $\mathcal{R}_i$ has a visibility zone of the form $g_{\mathbf{a}_i}^{-1}([0,d])$ where d is the scope. Our contribution is to show that characterizing the secure zone translates into a set-membership set estimation problem [**2**] where $\mathbf{x}(t)$ is shown to be inside the set $\mathbb{X}(t)$ returned by our set-membership observer. Then we conclude that $\mathbf{x}(t)$ cannot be inside the *complementary* of $\mathbb{X}(t)$. This result can be formalized by the following theorem.

Theorem. The virtual intruder has a state vector $\mathbf{x}(t)$ inside the set

$$\mathbb{X}(t) = \mathbb{O} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{O}$. As a consequence, the secure zone is

$$\mathbb{S}(t) = \overline{proj_{world}(\mathbb{X}(t))}.$$

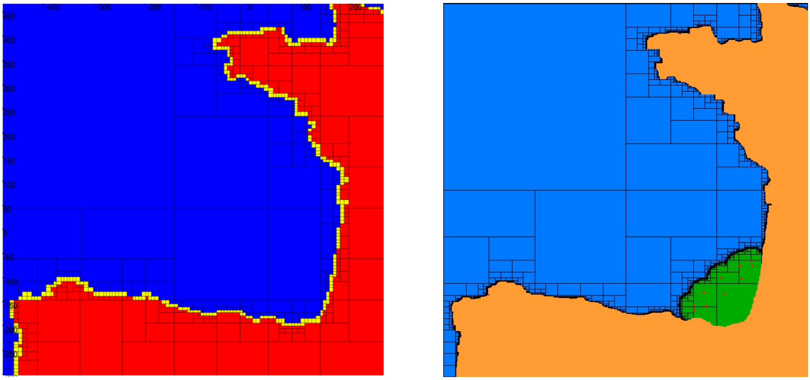

Figure 1: (left) Paving of the Biscay Bay, (right) Green: Secured zone

Proof (sketch). Two cases should be considered. If no actual intruder exists then $\mathbb{S}(t)$ cannot is secured. If the virtual intruder is a real one, its state $\mathbf{x}(t)$ is inside $\mathbb{X}(t)$ and its position (which is a part of the state) is inside $proj_{world}(\mathbb{X}(t))$. In both situations, the intruder can not be inside $\mathbb{S}(t)$.

Method. Each robot follows a reference point. All reference points form a flat ellipsoid which plays the role of a barrier. The strategy is illustrated by Figure 1 for 10 robots. The set $\mathbb{O}$ corresponds to the blue area (left). On the right, $\mathbb{S}(t)$ is painted green. The observer has been implemented using interval analysis.

References:

[**1**] L. Jaulin, *Mobile robotics*, ISTE editions, London, 2016.

[**2**] M. Kieffer, L. Jaulin, E. Walter, D. Meizel, Nonlinear Identification Based on Unreliable Priors and Data, with Application to Robot Localization, *Robustness in Identification and Control*, 3 (1999), LNCIS 245, pp. 190–203.